Appendix E

Basic Phasor Calculations

This appendix relates to Chapter 6, §6.3.

The relations that govern modulation of one sine wave by another are very simple:

$$\sin(x)\sin(y) = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)] \tag{1A}$$

$$\sin(x)\cos(y) = \frac{1}{2}[\sin(x-y) + \sin(x+y)] \tag{1B}$$

Conventional electronic transducers draw upon the above relationships rather directly. While the implementation details (and the filtering) vary among specific devices, all of these can be regarded as “algebraic” or “point on wave” transducers.

Rms quantities can also be determined by decomposing the $i(t)$ and $v(t)$ signals into components that can recombined to sharply display the desired information. This usually requires correlation logic that “projects” the signals onto a set of reference waveforms. In a transducer these would generally be developed from:

$$\begin{aligned} S(t) &= \sin(\omega_R t) \\ C(t) &= \cos(\omega_R t) \end{aligned} \tag{2}$$

Reference frequency ω_R might be the nominal value of the actual frequency ω_n, or it might be an estimate for ω_n that is developed during the measurement process itself. For three-phase calculations $i(t)$ and $v(t)$ become vectors, as do the reference waveforms. In such case the latter are extended to include sine and cosine waveforms displaced by $\pm 2\pi/3$ radians.

The concepts used here are exactly those underlying Fourier analysis and the demodulation of AM radio signals. The basic principles can be demonstrated through single-phase calculations. Define

$$\begin{aligned} i_s(t) &= \langle i(\tau), S(\tau)\rangle_w \equiv \mathrm{Avg}_w\{i(\tau)S(\tau)\} \\ i_c(t) &= \langle i(\tau), C(\tau)\rangle_w \equiv \mathrm{Avg}_w\{i(\tau)C(\tau)\} \end{aligned} \tag{3}$$

Here $\langle \bullet, \bullet \rangle_w$ indicates the projection of one signal onto another, and $\mathrm{Avg}_W\{\bullet\}$ indicates an averaging process. Both operations are performed with τ ranging across some window W. This window defines the length of the averaging process, the data to be processed at time t, and the filter weights to be used in forming the average.

Next consider the case

$$\omega_R = \omega_n \tag{4}$$

$$i(t) = i_0(t) = I_0 \sin(\omega_n t + \theta) \tag{5}$$

for which

$$
\begin{aligned}
i(t)S(t) &= I_0 \sin(\omega_n t - \varphi)\sin(\omega_n t) \\
&= \tfrac{1}{2} I_0 \left[\cos(\theta) - \cos(2\omega_n t + \theta)\right]
\end{aligned}
\tag{6}
$$

Suppose too that W produces an exact average across one cycle of S(t) and introduces a scale factor of 2. Then

$$
\begin{aligned}
i_s(t) &= \mathrm{Avg}_w\{I(t)S(t)\} = I_0 \cos(\theta) \\
i_c(t) &= \mathrm{Avg}_w\{I(t)C(t)\} = I_0 \sin(\theta)
\end{aligned}
\tag{7}
$$

Now $i(t)$ can be described by the complex variable

$$
\vec{i}(t) = i_s(t) + j i_c(t) = I_0[\sin(\theta) + j\cos(\theta)] = I_0\angle\theta \tag{8}
$$

where $j = \sqrt{-1}$. In power system applications $\vec{i}(t)$ is usually termed a *phasor*, or more specifically, a current phasor. The associated voltage phasor $\vec{v}(t)$ can be formed in a similar manner, and combined with $\vec{i}(t)$ to determine other quantities such as real or reactive power.

The example here rather idealized, in that ω_R exactly equaled ω_n and the averaging process exactly eliminated the double frequency term at $2\omega_n$. A practical instrument must deliver good performance without such exactitude. This requires close attention to signal processing details.